Analysis. Jan 2012
Some interesting problems.

Use the various problem-solving strategies to solve each problem. In many cases there is more than one possible approach, so be creative.

1. ***Catwoman's Cats*** If you ask Batman's nemesis, Catwoman, how many cats she has, she answers with a riddle: "Five-sixths of my cats plus seven." How many cats does Catwoman have? (April 20, 2003)

2. ***Pencil Collection*** Bob gave four-fifths of his pencils to Barbara, then he gave two-thirds of the remaining pencils to Bonnie. If he ended up with ten pencils for himself, with how many did he start? (October 12, 2003)

3. ***Adding Gasoline*** The gasoline gauge on a van initially read $\frac{1}{8}$ full. When 15 gallons were added to the tank, the gauge read $\frac{3}{4}$ full. How many more gallons are needed to fill the tank? (November 25, 2004)

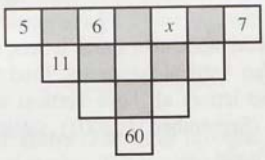
4. ***Gasoline Tank Capacity*** When 6 gallons of gasoline are put into a car's tank, the indicator goes from $\frac{1}{4}$ of a tank to $\frac{5}{8}$. What is the total capacity of the gasoline tank? (February 21, 2004)

5. ***Number Pattern*** What is the relationship between the rows of numbers?

18,	38,	24,	46,	42
8,	24,	8,	24,	8

(May 26, 2005)

6. ***Unknown Number*** The number in an unshaded square is obtained by adding the numbers connected with it from the row above. (The 11 is one such number.) What is the value of x? (August 9, 2004)



7. ***Locking Boxes*** You and I each have one lock and a corresponding key. I want to mail you a box with a ring in it, but any box that is not locked will be emptied before it reaches its recipient. How can I safely send you the ring? (Note that you and I each have keys to our own lock but not to the other lock.) (May 4, 2004)

8. ***Woodchuck Chucking Wood*** Nine woodchucks can chuck eight pieces of wood in 3 hours. How much wood can a woodchuck chuck in 1 hour? (May 24, 2004)

9. ***Number in a Sequence*** In the sequence 16, 80, 48, 64, A, B, C, D, each term beyond the second term is the arithmetic mean (average) of the two previous terms. What is the value of D? (April 26, 2004)

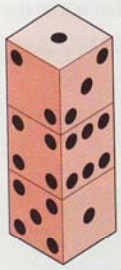
10. ***Unknown Number*** Cindy was asked by her teacher to subtract 3 from a certain number and then divide the result by 9. Instead, she subtracted 9 and then divided the result by 3, giving an answer of 43. What would her answer have been if she had worked the problem correctly? (September 3, 2004)

11. ***Labeling Boxes*** You are working in a store that has been very careless with the stock. Three boxes of socks are each incorrectly labeled. The labels say *red socks*, *green socks*, and *red and green socks*. How can you relabel the boxes correctly by taking only one sock out of one box, without looking inside the boxes? (October 22, 2001)

12. ***Vertical Symmetry in States' Names*** (If a vertical line is drawn through the center of a figure and the left and right sides are reflections of each other across this line, the figure is said to have vertical symmetry.) When spelled with all capital letters, each letter in HAWAII has vertical symmetry. Find the name of a state whose letters all have vertical and horizontal symmetry. (September 11, 2001)

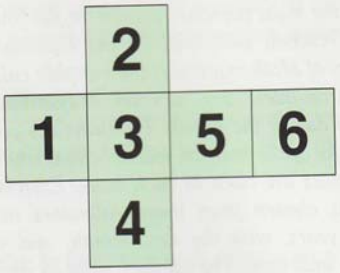
13. ***Sum of Hidden Dots on Dice*** Three dice with faces numbered 1 through 6 are stacked as shown. Seven of the eighteen faces are visible, leaving eleven faces hidden on the back, on the bottom, and between dice. The total number of dots not visible in this view is

A. 21
B. 22
C. 31
D. 41
E. 53
(September 17, 2001)

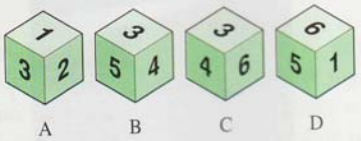


14. ***Mr. Green's Age*** At his birthday party, Mr. Green would not directly tell how old he was. He said, "If you add the year of my birth to this year, subtract the year of my tenth birthday and the year of my fiftieth birthday, and then add my present age, the result is eighty." How old was Mr Green? (December 14, 1997)

15. ***Unfolding and Folding a Box*** An unfolded box is shown below.

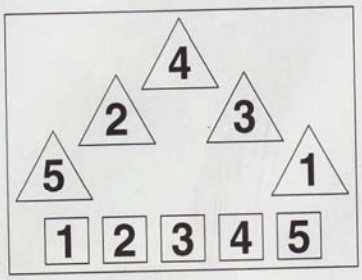


Which figure shows the box folded up? (November 7, 2001)

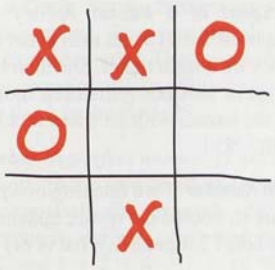


16. ***Age of the Bus Driver*** Today is your first day driving a city bus. When you leave downtown, you have twenty-three passengers. At the first stop, three people exit and five people get on the bus. At the second stop, eleven people exit and eight people get on the bus. At the third stop, five people exit and ten people get on. How old is the bus driver? (April 1, 2002)

17. ***Matching Triangles and Squares*** How can you connect each square with the triangle that has the same number? Lines cannot cross, enter a square or triangle, or go outside the diagram. (October 15, 1999)



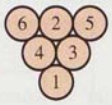
18. ***Ticktacktoe Strategy*** You and a friend are playing ticktacktoe, where three in a row loses. (See the next page.) You are O. If you want to win, what must your next move be? (October 21, 2001)



19. ***Forming Perfect Square Sums*** How must one place the integers from 1 to 15 in each of the spaces below in such a way that no number is repeated and the sum of the numbers in any two consecutive spaces is a perfect square? (November 11, 2001)

20. ***How Old?*** Pat and Chris have the same birthday. Pat is twice as old as Chris was when Pat was as old as Chris is now. If Pat is now 24 years old, how old is Chris? (December 3, 2001)

21. ***Difference Triangle*** Balls numbered 1 through 6 are arranged in a *difference triangle*. Note that in any row, the difference between the larger and the smaller of two successive balls is the number of the ball that appears below them. Arrange balls numbered 1 through 10 in a *difference triangle*. (May 6, 1998)



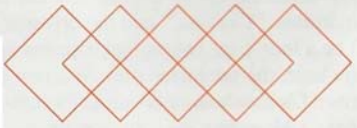
22. ***Clock Face*** By drawing two straight lines, divide the face of a clock into three regions such that the numbers in the regions have the same total. (October 28, 1998)

24. ***Perfect Square*** Only one of these numbers is a perfect square. Which one is it? (October 8, 1997)

329476 389372 964328
326047 724203

25. ***Sleeping on the Way to Grandma's House*** While traveling to his grandmother's for Christmas, George fell asleep halfway through the journey. When he awoke, he still had to travel half the distance that he had traveled while sleeping. For what part of the entire journey had he been asleep? (December 25, 1998)

26. ***Counting Puzzle (Rectangles)*** How many rectangles of any size are in the figure shown? (September 10, 2001)

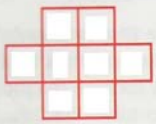


27. ***Buckets of Water*** You have brought two unmarked buckets to a stream. The buckets hold 7 gallons and 3 gallons of water, respectively. How can you obtain exactly 5 gallons of water to take home? (October 19, 1997)

28. ***Collecting Acorns*** Chipper and Dalie collected thirty-two acorns on Monday and stored them with their acorn supply. After Chipper fell asleep, Dalie ate half the acorns. This pattern continued through Friday night, with thirty-two acorns being added and half the supply being eaten. On Saturday morning, Chipper counted the acorns and found that they had only thirty-five. How many acorns had they started with on Monday morning? (March 12, 1997)

29. ***Counting Puzzle (Rectangles)*** How many rectangles are in the figure? (March 27, 1997)

30. ***Digit Puzzle*** Place each of the digits 1, 2, 3, 4, 5, 6, 7, and 8 in separate boxes so that boxes that share common corners do not contain successive digits. (November 29, 1997)



31. ***Palindromic Number*** (*Note:* A *palindromic* number is a number whose digits read the same left to right as right to left. For example, 383, 12321, and 9876789 are palindromic.) The odometer of the family car read 15951 when the driver noticed that the number was palindromic. "Curious," said the driver to herself. "It will be a long time before that happens again." But 2 hours later, the odometer showed a new palindromic number. (*Author's note:* Assume it was the next possible one.) How fast was the car driving in those 2 hours? (December 26, 1998)

32. ***Exchange Rate*** An island has no currency; it instead has the following exchange rate:

50 bananas = 20 coconuts
30 coconuts = 12 fish
100 fish = 1 hammock

How many bananas equal 1 hammock? (April 16, 1998)

33. ***Final Digits of a Power of 7*** What are the final two digits of 7^{1997}? (November 29, 1997)

34. ***Brightness of a Clock Display*** If a digital clock is the only light in an otherwise totally dark room, when will the room be darkest? Brightest? (May 1, 1996)

35. ***Value of Coins*** Which is worth more, a kilogram of \$10 gold pieces or half a kilogram of \$20 gold pieces? (March 20, 1995)

36. ***Units Digit of a Power of 3*** If you raise 3 to the 324th power, what is the units digit of the result?

37. ***Units Digit of a Power of 7*** What is the units digit in 7^{491}?

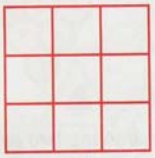
38. ***Money Spent at a Bazaar*** Ashley O'Shaughnessy bought a book for \$10 and then spent half her remaining money on a train ticket. She then bought lunch for \$4 and spent half her remaining money at a bazaar. She left the bazaar with \$8. How much money did she start with?

39. ***Unknown Number*** I am thinking of a positive number. If I square it, double the result, take half of that result, and then add 12, I get 37. What is my number?

40. ***Frog Climbing up a Well*** A frog is at the bottom of a 20-foot well. Each day it crawls up 4 feet, but each night it slips back 3 feet. After how many days will the frog reach the top of the well?

41. ***Matching Socks*** A drawer contains 20 black socks and 20 white socks. If the light is off and you reach into the drawer to get your socks, what is the minimum number of socks you must pull out in order to be sure that you have a matching pair?

42. ***Counting Puzzle (Squares)*** How many squares are in the following figure?



43. ***Counting Puzzle (Triangles)*** How many triangles are in the following figure?

44. *Children in a Circle* Some children are standing in a circular arrangement. They are evenly spaced and marked in numerical order. The fourth child is standing directly opposite the twelfth child. How many children are there in the circle?

45. *Perfect Number* A *perfect number* is a counting number that is equal to the sum of all its counting number divisors except itself. For example, 28 is a perfect number because its divisors other than itself are 1, 2, 4, 7, and 14, and $1 + 2 + 4 + 7 + 14 = 28$. What is the least perfect number?

46. *Naming Children* Becky's mother has three daughters. She named her first daughter Penny and her second daughter Nichole. What did she name her third daughter?

47. *Growth of a Lily Pad* A lily pad grows so that each day it doubles its size. On the twentieth day of its life, it completely covers a pond. On what day was the pond half covered?

48. *Interesting Property of a Sentence* Comment on an interesting property of this sentence: "A man, a plan, a canal, Panama."

49. *High School Graduation Year of Author* One of the authors of this book graduated from high school in the year that satisfies these conditions: (1) The sum of the digits is 23; (2) The hundreds digit is 3 more than the tens digit; (3) No digit is an 8. In what year did he graduate?

50. *Relative Heights* Donna is taller than David but shorter than Bill. Dan is shorter than Bob. What is the first letter in the name of the tallest person?

51. *Adam and Eve's Assets* Eve said to Adam, "If you give me one dollar, then we will have the same amount of money." Adam then replied, "Eve, if you give me one dollar, I will have double the amount of money you are left with." How much does each have?

52. *Missing Digits Puzzle* In the addition problem at the top of the next column, some digits are missing as indicated by the blanks. If the problem is done correctly, what is the sum of the missing digits?

$$\begin{array}{r} \_\,3\,5 \\ 8\,\_\,6 \\ +\ 1\,4\,\_ \\ \hline \_\,4\,0\,8 \end{array}$$

53. *Missing Digits Puzzle* Fill in the blanks so that the multiplication problem below uses all digits 0, 1, 2, 3, . . . , 9 exactly once, and is correctly worked.

$$\begin{array}{r} \_\,0\,2 \\ \times\ \ 3\,.\,\_ \\ \hline \_\,5,\_\,\_\,\_ \end{array}$$

54. *Magic Square* A *magic square* is a square array of numbers that has the property that the sum of the numbers in any row, column, or diagonal is the same. Fill in the square below so that it becomes a magic square, and all digits 1, 2, 3, . . . , 9 are used exactly once.

6		8
	5	
		4

55. *Magic Square* Refer to Exercise 54. Complete the magic square below so that all counting numbers 1, 2, 3, . . . ,16 are used exactly once, and the sum in each row, column, or diagonal is 34.

6			9
	15		14
11		10	
16		13	

56. *Paying for a Mint* Brian Altobello has an unlimited number of cents (pennies), nickels, and dimes. In how many different ways can he pay 15¢ for a chocolate mint? (For example, one way is 1 dime and 5 pennies.)

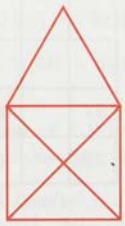
57. *Pitches in a Baseball Game* What is the minimum number of pitches that a baseball player who pitches a complete game can make in a regulation 9-inning baseball game?

58. ***Weighing Coins*** You have eight coins. Seven are genuine and one is a fake, which weighs a little less than the other seven. You have a balance scale, which you may use only three times. Tell how to locate the bad coin in three weighings. (Then show how to detect the bad coin in only *two* weighings.)

59. ***Geometry Puzzle*** When the diagram shown is folded to form a cube, what letter is opposite the face marked Z?

60. ***Picture Puzzle*** Draw a square in the following figure so that no two cats share the same region.

61. ***Geometry Puzzle*** Draw the following figure without picking up your pencil from the paper and without tracing over a line you have already drawn.



62. ***Geometry Puzzle*** Repeat Exercise 61 for this figure.

63. ***Decimal Digit*** What is the 100th digit in the decimal representation for $\frac{1}{7}$?

64. ***Books on a Shelf*** Volumes 1 and 2 of *The Complete Works of Wally Smart* are standing in numerical order from left to right on your bookshelf. Volume 1 has 450 pages and Volume 2 has 475 pages. Excluding the covers, how many pages are between page 1 of Volume 1 and page 475 of Volume 2?

65. ***Oh Brother!*** The brother of the chief executive officer (CEO) of a major industrial firm died. The man who died had no brother. How is this possible?

66. ***Teenager's Age*** A teenager's age increased by 2 gives a perfect square. Her age decreased by 10 gives the square root of that perfect square. She is 5 years older than her brother. How old is her brother?

67. ***Ages*** James, Dan, Jessica, and Cathy form a pair of married couples. Their ages are 36, 31, 30, and 29. Jessica is married to the oldest person in the group. James is older than Jessica but younger than Cathy. Who is married to whom, and what are their ages?

68. ***Making Change*** In how many different ways can you make change for a half dollar using currently minted U.S. coins, if cents (pennies) are not allowed?

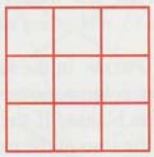
69. ***Days in a Month*** Some months have 30 days and some have 31 days. How many months have 28 days?

70. ***Dirt in a Hole*** How much dirt is there in a cubical hole, 6 feet on each side?

71. ***Fibonacci Property*** Refer to Example 1, and observe the sequence of numbers in color. Choose any four successive terms. Multiply the first one chosen by the fourth; then multiply the two middle terms. Repeat this process. What do you notice when the two products are compared?

72. ***Palindromic Greeting*** The first man introduced himself to the first woman with a brief "palindromic" greeting. What was the greeting? (*Hint:* See Exercises 31, 48, and 51.)

73. ***Geometry Puzzle*** What is the maximum number of small squares in which we may place crosses (×) and not have any row, column, or diagonal completely filled with crosses? Illustrate your answer.



74. ***Determining Operations*** Place one of the arithmetic operations +, −, ×, or ÷ between each pair of successive numbers on the left side of this equation to make it true. Any operation may be used more than once or not at all. Use parentheses as necessary.

1 2 3 4 5 6 7 8 9 = 100