Practice Block Parabolas

Activity 1: Parabolas (vertex form)

- *Time frame: 10 min*
- *Overview*
  - *Discuss the vertex form of a parabola*
  - *Guided construction of sliders to graph a vertex form of a parabolic function*

Discussion:

- In general what is the factored form of a parabolic function?
- How do the values affect the graph of the function?

Guided construction of sliders to graph a vertex form of a parabolic function

- Preparations
  - Open new GeoGebra file
  - Show algebra window
  - Show coordinate axes
  - Show grid (optional)

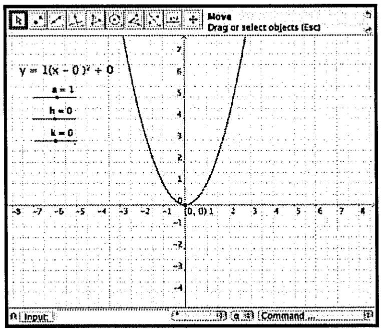
- Construction steps

1	a=2	Make three sliders named: *a, h,* and *k*
2		In the input field enter: f(x)=a (x – h)^2+k
3	ABC	Enter into text box: "f(x)="+a+"(x-"+h+")^2+"+k Red: represents the static text (without the quotation marks)

- Check construction
  - Drag the sliders and observe the changes in the graph, equation (algebra window), and the dynamic text.

- Properties
  - Change the color of the function and the text to match
  - Change font size and style
  - Change style of curve (thickness, solid, dashed, etc...)

Exeter Computer Lab – Parabolas –Vertex Form



After starting the program GeoGebra, choose Open under the File menu, and choose the file ***Parabvtx4.ggb***. When the file opens up, you should see a picture of a parabola on an axis system that looks like the one at right. The three sliders are used to change the values of *a*, *h*, and *k* in the equation $y = a(x-h)^2 + k$.

The diagram will change dynamically as you change the value of the three parameters.

1. To change the value for any of the three parameters, simply click on the point labeled *a*, *h*, or *k*, and then slide it back and forth on the horizontal line containing that point. Try it by changing the value of *a* while leaving *h* and *k* at 0.0.

a) What happens to the curve when the value of *a* increases so that it is greater than one?

b) What happens to the curve when the value of *a* decreases, but stays positive?

c) What happens to the curve when the value of *a* becomes negative?

d) What happens to the curve when the value of *a* becomes zero?

2. Change the value of *h* by dragging on the appropriate slider.

a) What happens to the curve when the value of *h* changes? Describe in reasonable detail.

b) Change the *h* and *a* sliders so as to display the parabola with vertex (2,0) and *y*-intercept (0,8). Write down an equation for this parabola in vertex form.

c) Use the sliders to display the parabola with the same shape as the one in b) above, but with vertex at (-4, 0). Write an equation of this parabola, and determine its *y*-intercept algebraically.

d) Use the *a* and *h* sliders to draw the parabola opening down with the same shape as the ones in b) and c), and whose vertex is at (2, 0). Write an equation of this parabola.

3. Change the value of *k* by dragging on the appropriate slider.

a) What happens to the curve when *k* changes? Describe in reasonable detail.

b) Change the *h, k,* and *a* sliders so as to display the parabola with vertex (-1, 2) and *y*-intercept (0, 5). Write down an equation in vertex form for this parabola.

c) Use the sliders to display the parabola with the same shape as the one in b) above, but with vertex at (5, -2). Write down an equation of this parabola, and determine its *y*-intercept even though it may not be in view.

4. First determine the equation of the following parabolas in the space below. Then, adjust the sliders appropriately to check to see if you have the correct parabola.

a) Vertex (1, -1), *y*-intercept (0, 4).

b) Vertex (-4, -3), contains point (-2, -1).

c) Vertex (5, 2), contains point (1, -6).

Activity 2: Parabolas (factored form)

- *Time frame: 10 min*
- *Overview*
    - *Discuss the factored form of a parabola*
    - *Guided construction of sliders to graph a factored form of a parabolic function*

<u>Discussion:</u>

- In general what is the factored form of a parabolic function?
- How do the values affect the graph of the function?

<u>Guided construction of sliders to graph a factored form of a parabolic function</u>

- Preparations
    - Open new GeoGebra file
    - Show algebra window
    - Show coordinate axes
    - Show grid (optional)

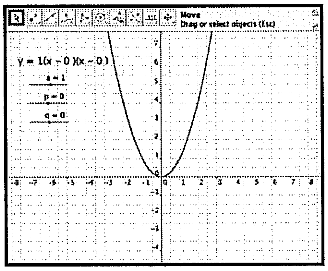
- Construction steps

1	a=2	Make three sliders named: *a, p,* and *q*
2		In the input field enter: f(x)=a (x-p) (x-q)
3	ABC	Enter into text box: “f(x)=”+a+”(x-“+p+”)(x-“+q+”)” Red: represents the static text (without the quotation marks)

- Check construction
    - Drag the sliders and observe the changes in the graph, equation (algebra window), and the dynamic text.

- Properties
    - Change the color of the function and the text to match
    - Change font size and style
    - Change style of curve (thickness, solid, dashed, etc…)

Exeter Computer Lab – Parabolas – Factored Form



After starting the program GeoGebra, choose Open under the File menu, and choose the file ***Parabfct4.ggb***. When the file opens up, you should see a picture of a parabola on an axis system that looks like the one at right. The three sliders are to be used to change the values of *a*, *p*, and *q* in the equation $y = a(x-p)(x-q)$. The diagram will change dynamically as you change the value of the three parameters.

1. To change the value for any of the three parameters, simply click on the point labeled *a*, *p*, or *q*, and then slide it back and forth on the horizontal line containing that point. Try it by changing the value of *a* while leaving *p* and *q* at 0.0.

a) What happens to the curve when the value of *a* increases so that it is greater than one?

b) What happens to the curve when the value of *a* decreases, but stays positive?

c) What happens to the curve when the value of *a* becomes negative?

d) What happens to the curve when the value of *a* becomes zero?

2. Change the value of *p* by dragging on the appropriate slider.

a) What happens to the curve when the value of *p* changes? Describe in reasonable detail.

b) Change the *p* and *a* sliders so as to display the parabola with *x*-intercepts at (0,0) and (4,0), and a vertex at (2, -8). Write down an equation in factored form for this parabola.

c) Use the sliders to display the parabola with the same shape as the one in b) above, but with x-intercepts at (0,0) and (-3,0). Write an equation of this parabola, and determine its vertex.

d) Use the a and p sliders to draw the parabola opening down with the same shape as the ones in b) and c), and whose x-intercepts are (0,0) and (6,0). Write an equation of this parabola, and determine its vertex.

3. Change the value of q by dragging on the appropriate slider.

a) What happens to the curve when q changes? Describe in reasonable detail.

b) Change the a, p, and q sliders so as to display the parabola with x-intercepts at (2, 0) and (-2, 0), with vertex at (0, 4). Write down an equation in factored form for this parabola.

c) Use the sliders to display the parabola with the same shape as the one in b) above, but with x-intercepts at (-3, 0) and (5, 0). Write down an equation of this parabola, and determine its vertex even though it may not be in view. Does the computer display give a perfectly accurate value for the vertex? Explain.

4. First determine the equation of the following parabolas in the space below. Then, adjust the sliders appropriately to check to see if you have the correct parabola.

a) x-intercepts at (-4, 0) and (6, 0), and Vertex (1, -5).

b) x-intercepts at (-5, 0) and (1, 0), and containing the point (3, 32)

c) x-intercepts at (0, 0) and (-3, 0), and containing the point (2, 3)

Activity 3: Parabola (standard-form)

Classification: Basic task

- *Time frame: 10 min*
- *Overview*
  - *Discuss the factored form of a parabola*
  - *Guided construction of sliders to graph a factored form of a parabolic function*

Discussion:

- In general what is the factored form of a parabolic function?
- How do the values affect the graph of the function?

Guided construction of sliders to graph a standard form of a parabolic function

- Preparations
  - Open new GeoGebra file
  - Show algebra window
  - Show coordinate axes
  - Show grid (optional)

- Construction process

  1. Make three sliders called *a*, *b*, and *c*

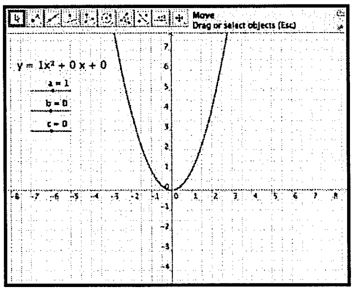
  2. Enter into the input field: $f(x) = ax^2 + bx + c$

  3. Insert dynamic text: "f(x) ="+a+"x^{2} +"+b+"x +"+c

- Check construction
  - Drag the sliders and observe the changes in the graph, equation (algebra window), and the dynamic text.

- Properties
  - Change the color of the function and the text to match
  - Change font size and style
  - Change style of curve (thickness, solid, dashed, etc...)

Exeter Computer Lab – Parabolas – Standard Form



After starting the program GeoGebra, choose Open under the File menu, and choose the file ***Parabstd.ggb***. When the file opens up, you should see a picture of a parabola on an axis system that looks like the one at right. The three sliders are to be used to change the values of *a*, *b*, and *c* in the equation $y = ax^2 + bx + c$. The diagram will change dynamically as you change the value of the three parameters.

1. To change the value for any of the three parameters, simply click on the point labeled *a*, *b*, or *c*, and then slide it back and forth on the horizontal line containing that point. Try it by changing the value of *a* while leaving *b* and *c* at 0.0.

a) What happens to the curve when the value of *a* increases so that it is greater than one?

b) What happens to the curve when the value of *a* decreases, but stays positive?

c) What happens to the curve when the value of *a* becomes negative?

d) What happens to the curve when the value of *a* becomes zero?

2. Change the value of *c* by dragging on the appropriate slider.

a) What happens to the curve when the value of *c* changes? Describe in reasonable detail.

b) Change the *c* and *a* sliders so as to display the parabola with a vertex at (0, 2) and an *x*-intercept of (1, 0). Write down an equation in standard form for this parabola.

c) Use the sliders to display the parabola with the same shape as the one in b) above, but with vertex at (0, 5). Write an equation of this parabola, in standard form.

d) Use the *a* and *c* sliders to draw the parabola opening up with the same shape as the ones in b) and c), and whose *x*-intercepts are (2, 0) and (-2, 0). Write an equation of this parabola, and determine its vertex even if it is not in your viewing window.

3. Change the value of *b* by dragging on the appropriate slider.

a) What happens to the curve when *b* changes? Describe in reasonable detail.

b) Change the *a, b,* and *c* sliders so as to display a parabola with axis of symmetry at $x = 2$ and passing through the origin. There are an infinite number of examples to choose from, write down an equation in standard form for two of them.

c) Use the sliders to display the parabola with the same shape as $y = 0.5x^2$, but with vertex at (-1, 4). Determine the x-intercepts for this parabola.

4. First determine the equation of the following parabolas in the space below. Then, adjust the sliders appropriately to check to see if you have the correct parabola.

a) *x*-intercepts at (-1, 0) and (3, 0), and Vertex (1, 2).

b) Vertex at (-2, 3), and contains the point (0, 11).

c) Contains the three points (0, -4), (1, -1), and (2, 1).

Activity 4: Geometric Construction of Parabola

- *Time frame: 20 min*
- *Overview*
  - *Discuss the geometric definition of a parabola*
  - *Guided construction of a parabola*

<u>Guided construction of a parabola</u>

- Preparations
  - Open new GeoGebra file
  - Show algebra window and hide coordinate axes if necessary
  - Change labeling setting to *New points only*

- Construction steps

1 Point A (*focus*)

2 Line (*directrix,* Line BC)

3 Point D on Line BC

4 Segment AD

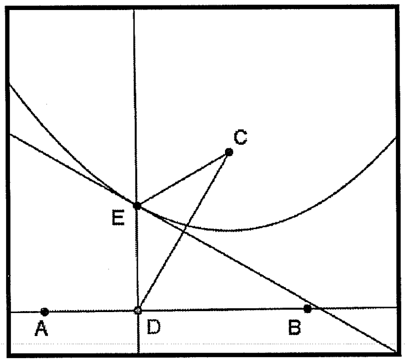
5 Perpendicularly bisect Segment AD

6 Thru Point D perpendicular to Line BC

6 Intersect the perpendicular bisector and perpendicular line to Line BC, Point E

7 Segment AE

8 Locus of E as D moves



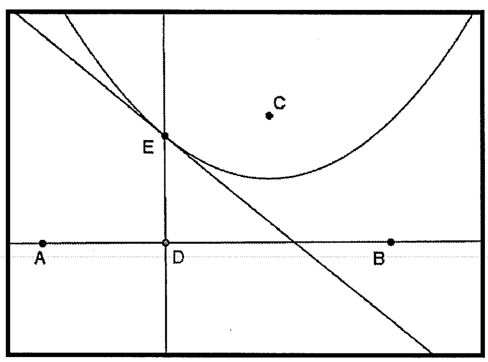
- Check construction
  - Move the focus and directrix to make sure the locus is a parabola
  - Show navigation bar to review construction step-by-step

- Discussion
    - How can we justify that Point E is equidistant from the directrix and the focus?
    - Discuss what happens when distance from the focus and directrix change.
    - Trace function before using the locus tool

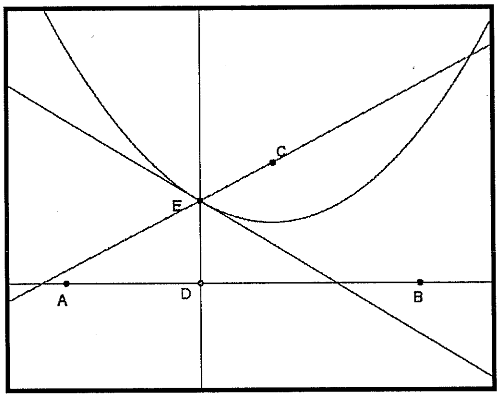
- To sketch the specific parabola whose *directrix* is $y = -1$, and whose *focus* is at (0,1), first show the axes and grid. Also, under the Options menu change point capturing to on (grid). It should now be easy to drag points A and B to locations on the line $y = -1$, and to drag the *focus* C to the location (0,1). Now move the focus to (0,2), and the directrix to $y = -2$? What do you notice happened to the equation*? Repeat this process for a couple of more examples and try and predict what is going on with the coefficient of the x-squared term in the equation, and the distance between the focus and the directrix.

**To view the equation we are going to have to actually construct the conic. Use the point tool and put 5 points on the locus. Then use the 'conic through five points' tool to construct the conic. Once you see the equation of the parabola click on the properties of the equation to change its form to 'y = ax^2+bx+c'. You may also have to change the decimal places in the Options menu.*

- How would you describe the relationship of the non-vertical line through E to the parabolic curve? If you said that this line is the *tangent line* to the curve, you are absolutely right. Let's examine the remarkable reflection property of parabolas that is the key to most of their everyday practical uses. First, hide the axes, grid, Segment DC, and Segment EC. Your picture should now look like the below.



- Reflect Line DE with respect to the tangent line. A new line is constructed, and for better contrast, change its color to red. Drag D back and forth and notice a very important property of this reflected line. It always passes through the *focus* of the parabola! Feel free to drag A, B, or C to different positions. Imagine that the line through D represents a light ray, a sound wave, or anything else. If it is traveling along a path perpendicular to the *directrix* of the parabola, it will reflect off the parabola to its *focus*. It is this important property that is used in satellite dishes, headlights in cars, and in many other applications.



Activity: Geometric Construction of an Ellipse

- *Time frame: 20 min*
- *Overview*
  - *Discuss the geometric definition of a parabola*
  - *Guided construction of a parabola*

Discussion:

- What is the geometric definition of an ellipse?

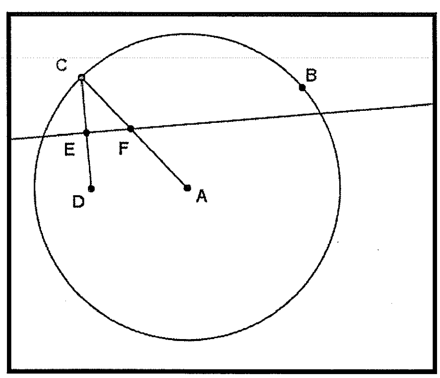
Guided construction of a parabola

- Preparations
  - Open new GeoGebra file
  - Show algebra window and hide coordinate axes if necessary
  - Change labeling setting to *New points only*

- Construction steps

1		Circle, centered at A containing B
2		Point C on Circle A & Point D inside the circle
3		Segment CD & Segment AC
4		Perpendicular Bisector of Segment CD
5		Points of intersection E and F between segment CD and the perpendicular bisector of CD and segment AC and the perpendicular bisector of CD

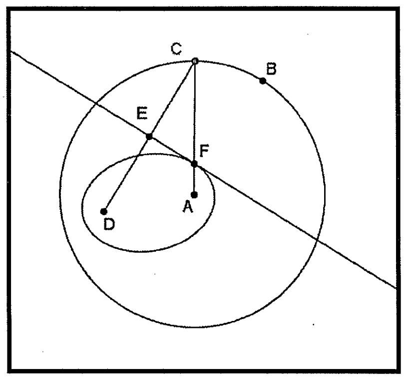
Your construction should look something like this.



Justification of the construction:

Distance FC equals FD by virtue of F being on the perpendicular bisector. Therefore, **AF + FC = AF + FD = radius of the circle AC**. This is true no matter what the position of C as it moves around the circle. Construct segment FD and make appropriate measurements to verify.

Right click on point F and select **Trace On**. Slowly drag C around the circle to see a temporary trace of the path of point F. Right click on F again and deselect **Trace On**, now to see a permanent path of Point F select the locus tool and click on Point F and then Point C.



The curve constructed is an *ellipse*. Points A and D are the *foci* of the ellipse, and the radius of the circle AC is the length of the major axis. Can you explain why?
Drag on A, B, C, and D to observe the changes in the diagram. Do you see the important property that the perpendicular bisector of CD has? Hopefully you have observed that this line is the tangent line to the ellipse at point F. To see a nice visual of this right click on the tangent line and select Trace On, observe what happens as Point C moves around the circle.

Ellipses have a special reflection property similar to parabolas that involves the foci. To demonstrate this reflection property, hide everything in the sketch but the ellipse, the tangent line, the foci A and D, and points F and C. Construct line AF. Reflect line AF using the tangent line as a line of reflection. Notice how the reflection always passes through the other focus D as you drag on point C and thus move F around the ellipse.