analysis 2.1.2013

Name: _____

Class: _____

Date: _____

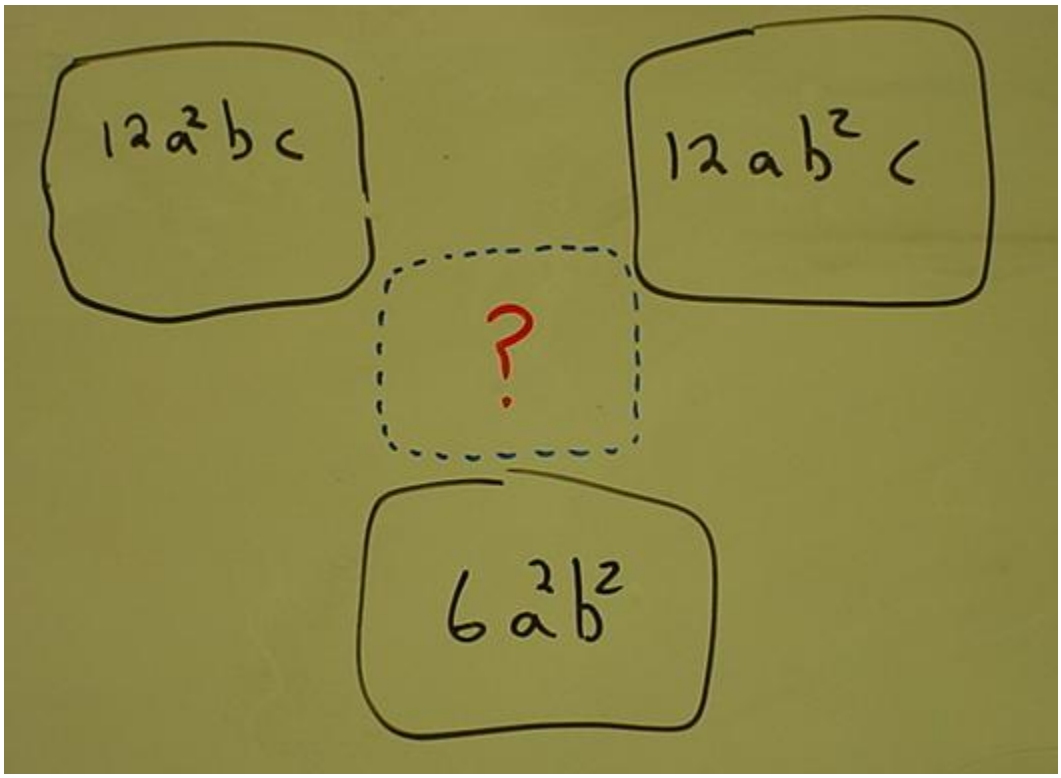
$(2a-b)^7$

1. Consider the expansion of the binomial in the attached image. How many terms will exist in this expansion?

2. How many 6 digit positive integers exist?

3. How many three digit positive integers start with an even digit, and end with an odd digit?

4. How many four digit positive integers exist



5. The black blobs in the attached image show three labeled marshmallows in some particular level of the pascal's structure we built in class. The dashed blue blob is the marshmallow in the next lower level. What will the coefficient of this marshmallow be?

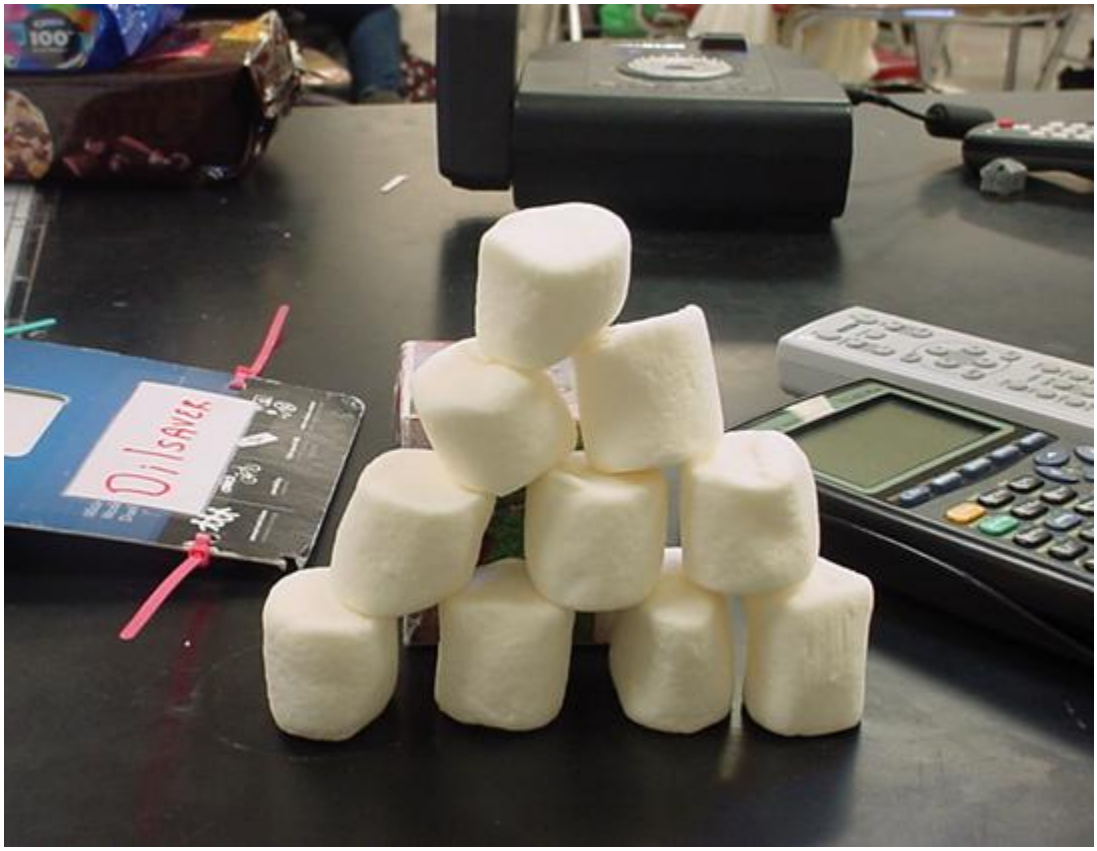
6. In the attached photo, the black blobs are marshmallows in some particular level of the structure we built in class using toothpicks and marshmallows. What is the name (number) of the level containing the marshmallows represented by these black blobs?

7. How many positive four digit numbers exist which have one or more 7's or 8's (it's ok to have both).

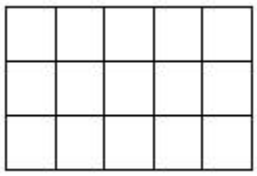
8. Suppose Hunter and Phillip decided to add more layers to the thing-that's-one-level-beyond-pascal's triangle that we built in class using toothpicks and marshmallows (or gummy bears, or dots). They build down to level 13. How many Marshmallows will Hunter and PhilliP need to use to build level 13 (yes, I'm talking about ONLY level 13)

$20a^3bc$

9. In the pascal structure that we built in class using toothpicks and marshmallows, if we had continued to build to level 5 one of the marshmallows would be labeled as shown in the attached image. If we think of the structure as representing a path problem, with each path starting at the top marshmallow and the only legal moves being down in either the "a", "b", or "c", directions, how many different paths go from the top marshmallow to the marshmallow labeled as shown in the attached image.

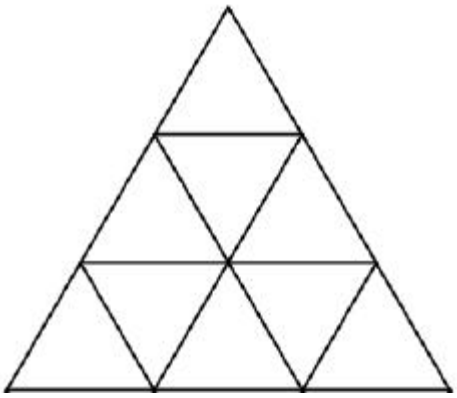


10. Young Brooke W was sitting eating breakfast at the Sand's cafe with MrW and MrD, building triangular structures out of coffee creamers as hinted at in the attached image. The image shows marshmallows instead of coffee creamers. Of course Brooke was energetic and kept flipping her coffee creamers one by one into Mr D's oatmeal. In self-defense, Mr D said to Brooke "If you had perfect balance so the stack wouldn't fall over and if you had a lot more coffee creamers, say 800, how tall would the stack be and how many creamers would be left over. If H is the number of layers of coffee creamers (the attached image has 4 layers) and R is the number of coffee creamers remaining, compute H + R.



11. Suppose that the cells that look like squares in the figure are actually squares. How many squares are in the figure, counting in the usual way that we do, following along any existing lines in the figure

A. 22
B. 23
C. 24
D. 25
E. 26
F. 27
G. None of these are correct



12. Consider the triangular grid shown in the attached image. Let's count the number of triangles of all different sizes formed by tracing along exising lines in the image.

13. Suppose Mr D's room contains 6 rows of student desks. Suppose further that the room is empty when 5 eager students walk in, each of whom insists on sitting in a front row seat. In how many different arrangements may these students take their front row seat?

A. 4!
B. 5!
C. 6!
D. 4
E. 5
F. 6
G. None of the other choices are correct.

14. Not-that-many-years-from-now, Phillip W Dunger will (may) be appointed director of transportation for the state of Missouri. One aspect of his job will (may) be to plan license plate production for motor vehicles. Phillip notes that the 2010 census of Missouri showed 5,987,580 residents. Phillip estimates that each resident has an average of 4 licensed cars (yes, for everyone). Phil's plan is to use 4 non-repeating letters followed by some number of non-repeating digits. If director Dunger uses this method to produce license plates for Missouri, what is the least numbers of digits he can use and have enough different license numbers for all the cars in Missouri?

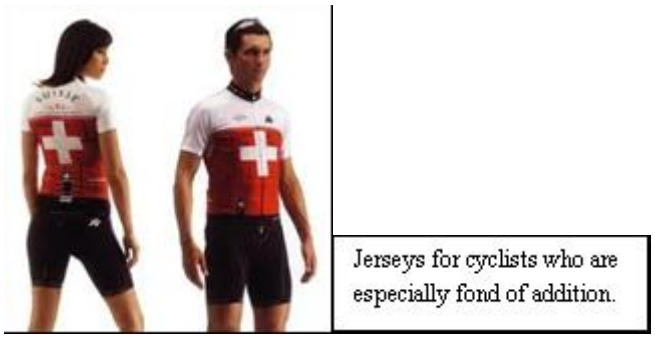
A. 0
B. 1
C. 2
D. 3
E. 4
F. 5
G. 6
H. 7
I. 8
J. 9

15. Let's agree that a "word" can be any sequence of letters. If we rearrange the letters of PASCALWASARASCAL, how many different 16 letter "words" can we make?

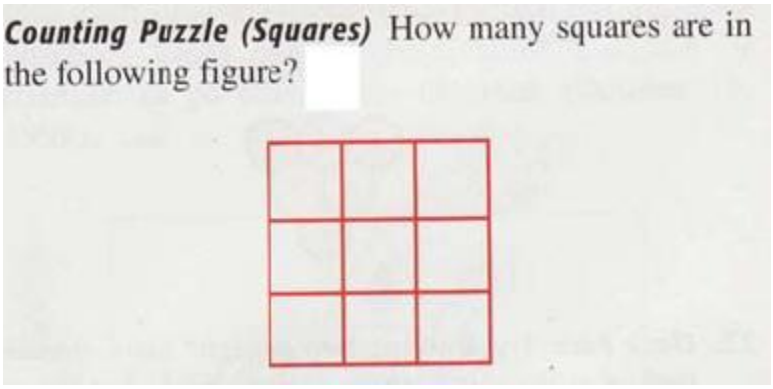
A. 1441440
B. $\frac{16!}{6!\cdot 3!\cdot 2!\cdot 2!}$
C. $\frac{16!}{6!\cdot 5!\cdot 3!\cdot 2!}$
D. 1,814,414,440
E. 1,800,101,441,440
F. None of the other choices are correct.

16. Nathan D is considering the digits 1, 3, 5, 7, 9. These are all the odd digits. He is thinking about making a 5 digit number, or maybe a 1 digit number using these 5 digits. How many different 1 digit or 5 digit numbers could Nathan make under these conditions?

17. Suppose a math teacher decides to relax by bicycling. Its summer and the temperature is higher than it is currently. The cyclist has a red spandex jersey, a yellow jersey, and a black wool jersey. (A jersey is another word for a shirt). The cyclist also has a pair of brown cycling shorts and a pair of dark blue cycling shorts. Lets suppose the cyclist is going to wear one jersey and one pair of shorts. In how many different ways can he use the clothes weve listed to dress for this ride?



A. 5
B. 3!2!
C. 5!
D. 6!
E. 6
F. 2^3
G. 3^2



18. As we've done on other problems, we're only counting squares whose sides lie on lines of the existing figure.

A. 8
B. 9
C. 10
D. 11
E. 12
F. 13
G. 14
H. 15

19. In how many ways can a 10 member club elect 3 officers, with the usual rule in place a person can hold at most one office.

A. ${}_{10}P_3$

B. ${}_{10}C_3$

C. 10^3

D. 3^{10}

E. none of these are correct

20. In how many zeros does the number 184! end?

A. 34

B. 44

C. 54

D. 64

E. 74

21. How many distinct 5 card hands of cards are possible?

A. ${}_{52}P_5$

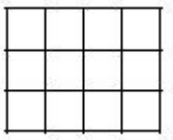
B. 52!

C. ${}_{52}C_5$

D. 52^5

E. 47!

22. By how many distinct paths may one get from the upper left corner to the lower right corner of the grid shown at the right provided the only legal moves are "right" or "down".



A. 21

B. 28

C. 35

D. 56

E. 84

Final Digits of a Power of 7 What are the final two digits of 7^{1997}? (November 29, 1997)

23. Ok, Hunter, will that wonderful TI-Inspire you're toting around so the 7 to-a-really-big power thing???

24. Suppose Tyler is working with the digits 2, 4, 5, 6. He is either going to make a four digit number or a two digit number using these digits, with no repetition. In how many different ways can Tyler either make a four digit number or a two digit number?

A. 8
B. 12
C. 24
D. 36
E. 288

Matching Socks A drawer contains 20 black socks and 20 white socks. If the light is off and you reach into the drawer to get your socks, what is the minimum number of socks you must pull out in order to be sure that you have a matching pair?

25. Ok, its a day one problem from the sheet. Be careful.

26. Suppose a tray has an abundant (this means more than 10 of each) supply of chocolate chip, peanut butter, sugar, and persimmon cookies. In how many different ways could Tyler select 10 cookies for your group from this tray?

27. Consider the letters MISSISSIPPI. In how many ways may these 11 letters be arranged into one row such that no two Ss are adjacent?

28. How many ordered triples of positive integers (x,y,z) exist such that x+y+z = 50?

29. Consider $(a+b+c+d+e)^8$. If this polynomial were multiplied out, how many terms would it contain?

30. Evaluate $\sin(\arctan(\frac{3}{4}))$. You may suppose the angle is in standard position and terminates in first quadrant.

A. $\frac{4}{3}$ B. $\frac{3}{5}$ C. $\frac{5}{4}$

D. $\frac{5}{3}$ E. $\frac{4}{5}$

31. Bonus. How many 4 digit numbers are divisible by 5 but not divisible by 3?

32. Convert 345° into radians.

A. $62{,}100\pi$ B. 1.92 C. $\frac{23\pi}{12}$ D. $\frac{12\pi}{23}$

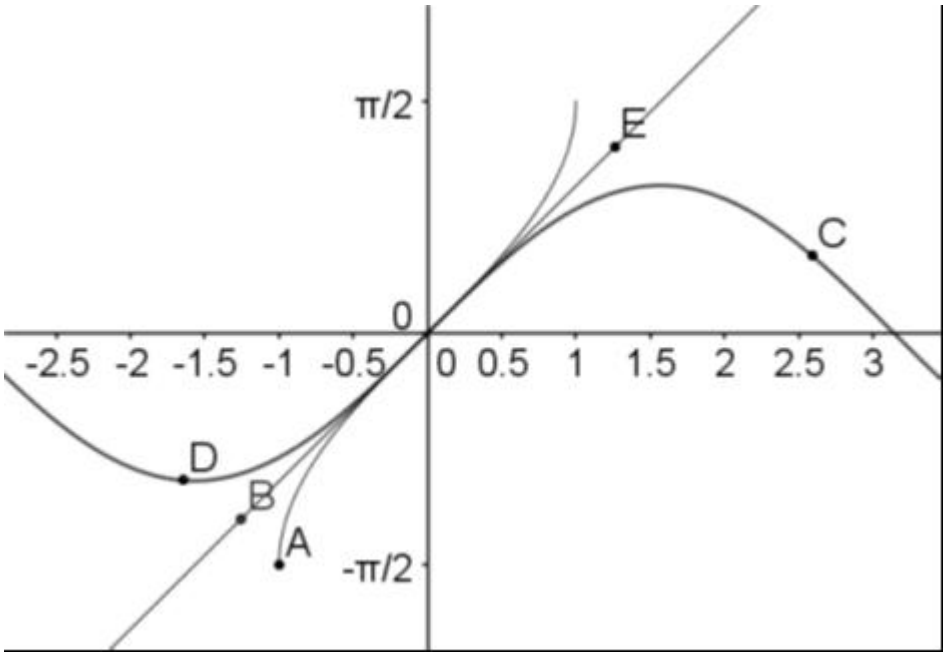
33. Perhaps with the aid of a calculator, evaluate csc(3.4 radians).

A. -3.91 B. -0.25554 C. 16.86

D. 0.0593 E. 1.0017

34. Find $\sin\theta$ when $\cos\theta = \frac{8}{17}$ *and* $\tan\theta < 0$

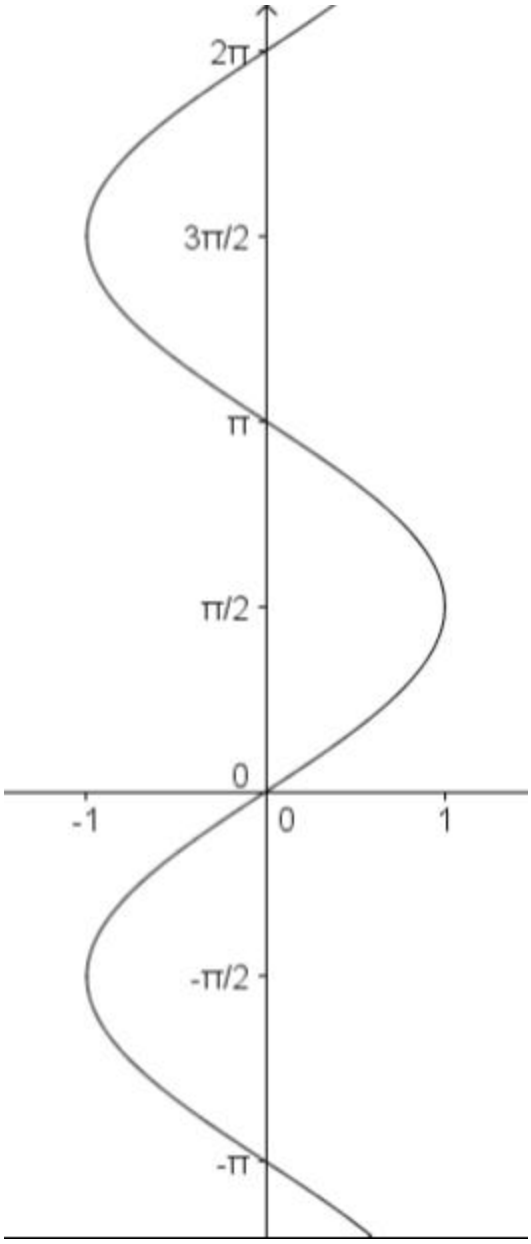
A. $\frac{-17}{15}$ B. $\frac{-15}{17}$ C. $\frac{15}{17}$ D. $\frac{17}{15}$



35. Consider the family of graphs shown above. Which point is plotted on the graph of $y = \sin^{-1}(x)$?

A. A B. B and E C. C

D. D E. D and C



36. The graph of the inverse relation for the sine function is shown above. This is the graph we drew on graph paper in pencil and folded along a certain reflection line and then traced to produce a graph of the inverse relation. To form a function we restricted the range of the inverse relation until it would pass the vertical line test. Which of the following gives the range of the function $y = \sin^{-1}(x)$.

A. $-1 \le y \le 1$

B. $\frac{\pi}{2} \le y \le \frac{3\pi}{2}$

C. $0 \le y \le \pi$

D. $\frac{3\pi}{2} \le y \le \frac{5\pi}{2}$

E. $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$

37. In the triangle shown at the right, find $\tan(\theta)$

A. $\frac{15}{8}$

B. $\frac{15}{17}$

C. $\frac{8}{15}$

D. $\frac{15}{17}$

E. $\frac{8}{17}$