Counting

A problem a day helps keep boredom away
Some more fun problems.

1. How many ordered triples of positive integers (x,y,z) exist such that x+y+z=88?

2. The cookie problem. Suppose a tray comes around to you which has an ample supply of chocolate chip, peanut butter, and oatmeal-raisin cookies. You are to select 8 cookies for your group. By an amply supply, it is meant that there are more than 8 of each type of cookie on the tray. In how many distinct ways may you select the 8 cookies?

3. How many ordered 6-tuples of positive integers exist such their sum is 10?

4. Consider $(a+b+c+d)^{10}$ if we were to multiply this out, how many different terms would the resulting polynomial contain?

5. Consider $(a+b+c+d)^{10}$ if we were to multiply this out, what is the sum of the coefficients of all the terms?

6. Consider $(a+b+c+d)^{10}$; somewhere in the expansion of this polynomial, there would be a term _____ $a^2b^6c^1d$— (I could have left the 1 off the exponent of c, it would mean the same). Fill in both blanks.

7. How many ordered triples of non-negative integers (x,y,z) exist such that x+y+z=10.

8. Ok poker players. (a) in how many ways may one be dealt 5 cards with 2 pair the same. (b) what is the probability of being dealt a two pair hand?

9. More cards. (a) in how many ways may one be dealt a full hand (three of one value of card, two of another) (b) what is the probability of being dealt a full hand?

10. Again in five card hands of cards, in how many ways may one be dealt four of a kind? and (b) what is the probability of being dealt four of a kind?

11. Consider the letters MISSISSIPPI. In how many ways may these letters be rearranged so that no arrangement contains any adjacent S's?

12. First note how many positive 4 digit numbers exist. _____ Now of all these, how many have four distinct digits that are either increasing (like 2569, or 4678) or decreasing (like 9721 or 7643)?

13. Eight friends, A, B, C, D, E, F, G, and H are going to the big Lady Gaga concert. The group has tickets for eight seats all together in one row. A and D are friendly and insist on sitting together. F and G are also a couple and insist on sitting together. No one else particularly cares how their seating is arranged. In how many distinct ways can they seat themselves in their 8 seats?

14. Consider $(a+b+c)^5$; how many terms are in the polynomial that results when this is multiplied out?

15. Consider $(a+b+c)^5$; what is the sum of the coefficients of the terms of the polynomial that results when this is multiplied out?

16. Consider $(a+b+c)^5$. How many different terms could be produced of the following form? _____ a^2b-c-. For each such term possible, fill in the coefficient and the exponents for b and c.

Bonus: Consider the triangle grid shown at the right. How many triangles are in the figure at the right, counting only triangles whose sides trace along existing line segments in the grid. Hint. It might be productive to consider smaller triangles and look for a pattern.